On Approximation of Reserve Factors Dependency on Loads for Composite Stiffened Panels

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Abstract. We present two level approach to build accurate approximations for Reserve Factors dependency on loads for composite stiffened panels. Such dependency is continuous non-smooth function with complex form plateaux regions (i.e. regions where function has zero gradient), defined on low dimensional grids. The main problem that arises if one tries to construct global approximation in such case is the occurrence of Gibbs effect (i.e. harmonic oscillations of prediction) near the borders of plateaux that may significantly deteriorate approximation quality. Viable existing solution: approximation based on linear triangular interpolation avoids oscillations, but unlike the proposed approach it provides model that is not smooth outside plateaux regions and generally requires larger sample size to achieve same accuracy of approximation.

Introduction

We consider the problem of building accurate approximation for Reserve Factors dependency on loads for composite stiffened panels. This dependency is continuous non-smooth function of the special form (see an example on fig. 1). It is smooth continuous in almost all input domain except of several regions of complex form where plateaux regions with high function values and zero gradients are located.

Functions of such form are of practical interest because they often appear in problems of strength analysis of aeronautical structures. Aeronautical structures are mainly made of stiffened panels, i.e. thin shells (also called skin) enforced with stiffeners in both orbital and longitudinal directions. These basic structures are subject to highly non-linear phenomena such as buckling, collapse and damage tolerance. The whole structure has to require some strength conditions to check it's reliability, and most of them could be formulated using reserve factors (RF) values: the structure is validated if all of its RF are greater than 1. RF values are usually obtained using computationally expensive Finite Element (FE) method. During the sizing process, i.e. optimizing geometry of structure with respect to certain criteria (usually mass minimizing), RF values are taken as optimization constraints that allow to conclude if considered geometry would be reliable. So for all basic structures RFs have to be recomputed on every optimization step becoming very expensive operation in terms of time.

In recent years number of attempts were made to make calculation of RFs faster than with FE method. Promising results [1, 2] were obtained with the use of so-called surrogate models (or meta models). Surrogate models provide a continuous and differentiable function that approximates solution of Finite Element method based on sample of precomputed RF values obtained with FE method.

RF value depends on geometry and on the forces applied to the structure. Here we would address the case when geometry is fixed and only dependency of RF on applied forces is considered. In such case number of input parameters is quite small. So full factorial experimental design is used, i.e. for each input parameter a set of values is used and all possible combinations of these values are included in the design. Such design is the best to ensure good coverage of input domain [3].

Due to a nature of FE solvers high RF values often can not be computed precisely. As optimization constraints are satisfied for these points exact RF values are not important so they are set equal to some big constant value. Such points with constant output form complex shape plateaux regions. As exact value of the output is not known in these regions we would refer to them as ``nan"-regions. Knowing the location of these ``nan"-regions and accurately approximating high gradients borders of them is important for optimization purposes.

In previous works [1, 2, 4] attempts were made to construct global smooth approximation for RFs on all input domain. The remaining problem with such approach, applied to the considered specific type of approximation problem, is that it leads to occurrence of Gibbs effect (i.e. harmonic oscillations of prediction) on borders of plateaux regions that deteriorates approximation quality and makes function more complex for optimization (see fig. 2).

In this work we propose an approach that for considered problem completely removes Gibbs effect at ``nan"-regions borders and at the same time provides smooth and accurate approximation outside the borders. The approach uses two level scheme. At the first level classifier predicts if the new point belongs to a ``nan"-region based on a nearest neighbours approach. At the second level smooth approximation model constructed using only points outside the ``nan"-region predicts the output value.

The article is organized as follows. In section 1 a statement of approximation problem is considered. In section 2 possible approaches to build approximation on full factorial design with holes are discussed. In section 3 the proposed two level approach is described in details. In section 4 results of application of the proposed methodology to approximate RFs of real stiffened panels are presented.

Fig. 1: An example of functions considered

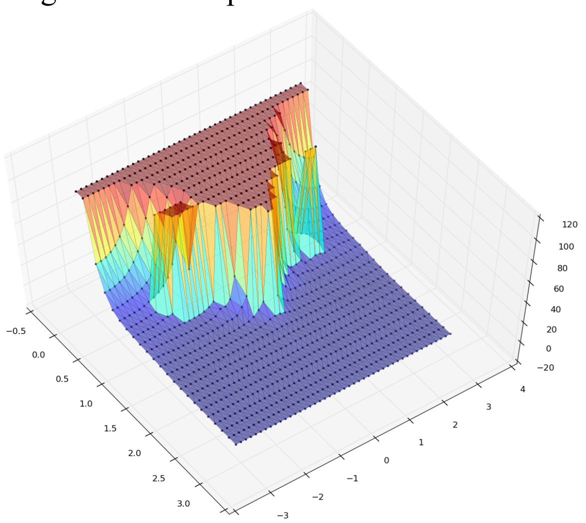
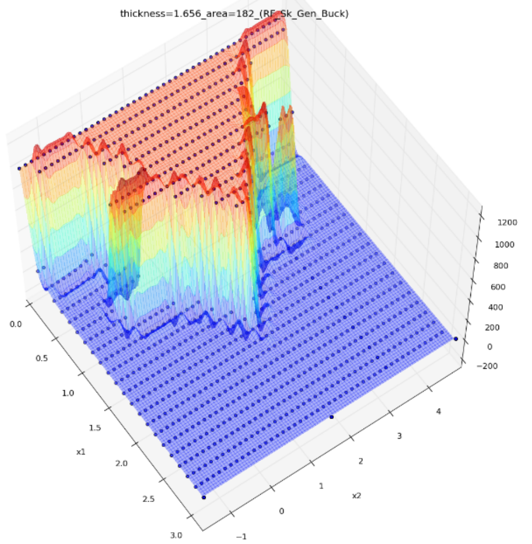


Fig. 2: Example of Gibbs effect



Considered problem statement

We consider a common surrogate model construction problem statement. The output is generated by a certain system which may be represented as a certain deterministic continuous function $f : \mathbf{x} \to y$. The sample $\mathbf{S} = \{\mathbf{X}, \mathbf{Y}\} = \{\mathbf{x}_i, y_i = f(\mathbf{x}_i), i = 1, \ldots, N\}$ of computed outputs is available. All $\mathbf{x}_i \in [-1, 1]^p$ and form full factorial grid design.

The problem is to construct an approximation $\tilde{f}(\mathbf{x}) = \tilde{f}(\mathbf{x}|\mathbf{S})$ using training sample $\mathbf{S}$, which is close to the $f(\mathbf{x})$ in a sense that empirical risk value $er_S(\tilde{f}) = \left(1/N \sum_{i=1}^{N} \left(f(\mathbf{x_i}) - \tilde{f}(\mathbf{x_i})\right)^2\right)^{1/2}$ is low. This error is very convenient to work with, however in practice engineers often want to know the worst case scenario results and errors like 95% quantile of the relative error are preferred:

$$er_S^{0.95}\left(\tilde{f}\right) = quantile_{0.95}\Big\{\big|\tilde{f}(\mathbf{x}_i) - f(\mathbf{x}_i)\big|/f(\mathbf{x}_i), i = 1, \ldots, N\Big\}. \quad (1)$$

Direct models parameters optimization with respect to latter error is rather complex, so basically surrogate models are constructed to minimize error $er_S(\tilde{f})$ while on model validation step error (1) is checked on independent test sample to decide if the model quality is satisfactory.

Techniques for factorial design

An approximation technique is based on an expansion in tensor product of given basis functions sets $\{\psi_{j_k}^k\}, k = 1, \ldots, p$ corresponding to each input dimension:

$$\tilde{f}(\mathbf{x}) = \sum_{j_1, j_2, \ldots, j_p} A_{j_1, j_2, \ldots, j_p} \psi_{j_1}^1(x^1)\psi_{j_2}^2(x^2)\ldots\psi_{j_p}^p(x^p). \quad (2)$$

where $\mathbf{x} = (x^1, \ldots, x^p)$ and A is an unknown tensor (multidimensional matrix) of coefficients.

Such model family include multivariate splines models and are highly recommended to use over other approximation techniques for it's good properties if data is noise free and an experimental design is full factorial [5]. Since all basis functions $\{\psi_{j_k}^k\}, k = 1, \ldots, p$ are given the function $\tilde{f}$ is parametrized only by the coefficients A. These coefficients can be found as a solution of optimization problem:

$$\frac{1}{N}\sum_{i=1}^{N}\left(y_i - \tilde{f}(\mathbf{x}_i)\right)^2 + \sum_{k=1}^{p}\lambda_k \left\|\frac{d^2\tilde{f}}{(dx^k)^2}\right\|^2 \underset{A}{\to} \min, \quad (3)$$

where $\|\cdot\|$ is the L_2 norm on $[-1, 1]^p$ and $\lambda_k > 0$ are regularization parameters. The penalty term limits redundant model oscillations which are measured using the second derivatives norm.

As $\mathbf{X}$ is a full factorial design it's possible to calculate the tensor A in a very efficient way. If some points of full factorial design are missing computation of A becomes more algorithmically complex but still efficient. Details on solution of (3) as well as the corresponding numerical algorithms are given in [6].

Two level approximation scheme

All points on input domain may be divided in two groups. Points that lie outside ``nan''-regions and points that lie inside. Points from the first group may be considered as generated by some smooth function and accurate smooth approximation may be built for them, for the second group it is even simpler as all points there lie on a constant value plateaux.

The reason that global smooth approximation techniques produce Gibbs effect is that because on the border of ``nan''-region discontinuity in the first order derivatives of f occurs as the approximation have to be close to points from both groups. So the idea is that Gibbs effect may be avoided if for each group of points a separate approximation is constructed and some classifier is available that for a new point tells which group the new point belongs to.

As $\mathbf{X}$ is a full factorial design, meaning uniform coverage of input space, classifier based on nearest neighbours approach [7] is an effective tool to determine which group a point belongs to. Another advantage of such classification scheme is that it would work even if there are several disjoint plateaux regions in the design space.

Fig. 3: An example of results obtained with proposed scheme for function from fig. 1

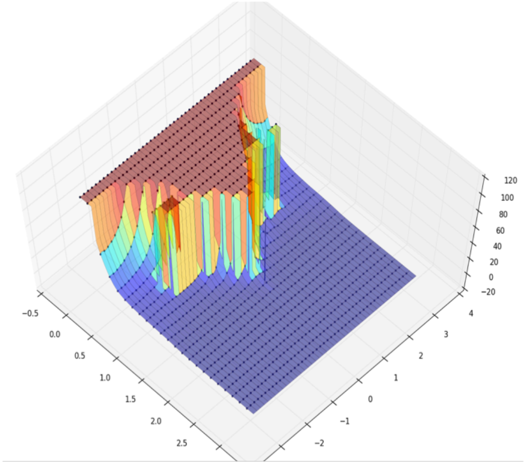
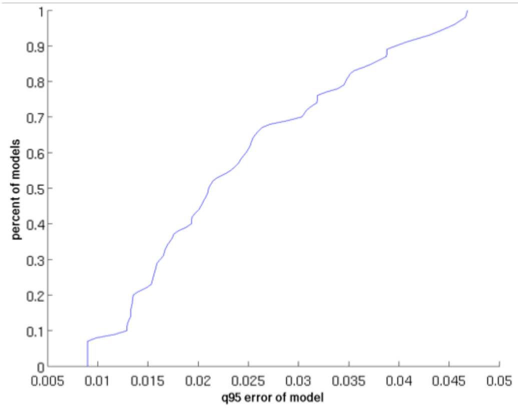


Fig. 4: Cumulative distribution function of $er_S^{0.95}\left(\tilde{f}\right)$ error for buckling Reserve Factors on 47 different geometries



Points of $\mathbf{X}$ belonging to the first group form a full factorial design with holes so the algorithm from (2), (3) may be used. Note that the proposed approach may be extended to a more general case of approximation problems where f contains discontinuities. In such cases points located on different sides of discontinuities should be assigned to different groups.

Such approach may be considered as a member of mixture of experts methods [8] family where non parametric rule is used to select expert (local approximation) and hard clustering scheme is used (i.e. only one expert is taken to form prediction in each point).

Application of the proposed scheme

The proposed approach was used to construct approximations for Reserve Factors of Composite Stiffened Panels for one catalogue from PRESTO database, belonging to Airbus company, and used for $A350-900$ Structural Development and Certification. Data consisted of 47 panels geometries for each of which full factorial design input sample of size $45 \times 23 = 1035$ was given. 7 types of Reserve Factors were considered. ``Nan"-regions were formed by Reserve Factors with output values ≥ 100 (which were set equal to 100 in the database). Thus, it was required to construct $47 \times 7 = 329$ approximations.

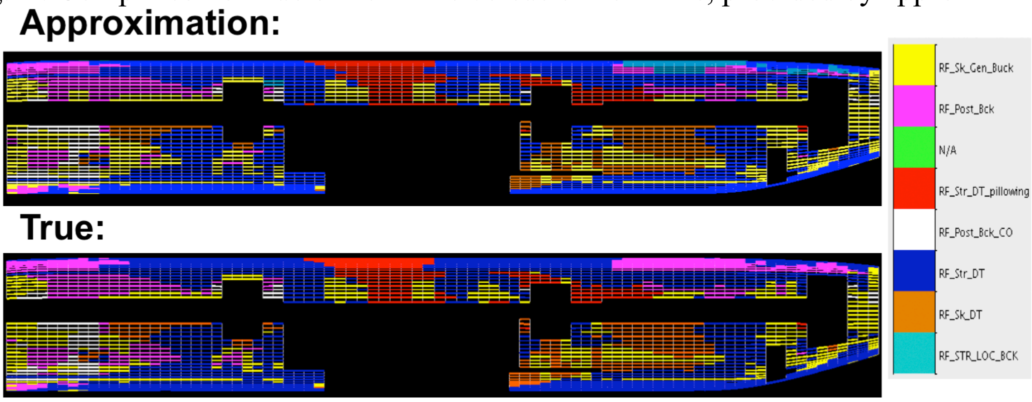
The proposed two-level scheme was used to construct all the models. The points were divided in 2 groups with output values less than 100 and equal to 100. For the first group the described approximation technique for factorial design was used to construct approximation. For the second group an output value 100 was assigned to all points. Following results were obtained:

- the size of constructed approximations was ~ 9 times smaller than the size of the training sample (very important property since the size of the initial database is a bottleneck in such problems),
- no Gibbs effect was present in any of the constructed models, see an example on fig. 3,
- it took about 2 minutes on conventional PC to construct all 329 models with the proposed method (and it takes about 10 seconds to make prediction for 100000 new input points),
- relative error is lower than 5% for of 95% constructed models.

An example of the error distribution for one buckling reserve factor for all geometries is shown on fig. 4. One may see that for all geometries $er_S^{0.95}\left(\tilde{f}\right)$ error is smaller than 0.05 which was concluded to be satisfactory accuracy by Airbus engineers.

Comparison of true critical Reserve Factors (i.e. RFs that are close to 1) versus approximation for found optimum design is drawn on fig. 5. One may see that approximation provides critical Reserve Factors very similar to the true ones.

Fig. 5: Comparison of true critical RFs versus critical RFs, provided by approximations



Summary

In the article the methodology was proposed that allows to remove Gibbs effect around plateaux regions in non-smooth continuous functions and obtain smooth approximation outside these regions. The methodology was applied to solve the practical problem of approximating Reserve Factors versus loads for stiffened panels, posed by Airbus company. Good results were obtained. With slight modification the methodology may be extended to handle more general class of functions with discontinuities that splits input domain into several smooth regions.

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